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Its axis is parallel to the X-axis and its semi-axis is $\sqrt{(a^2-b^2)}$. Since its eccentricity is $\sqrt{2}$, the distance of its foci from the center is $\sqrt{[2(a^2-b^2)]}$, and that of its directrices is $\frac{1}{2}\sqrt{[2(a^2-b^2)]}$. It may be neatly constructed as follows (Fig. 2):

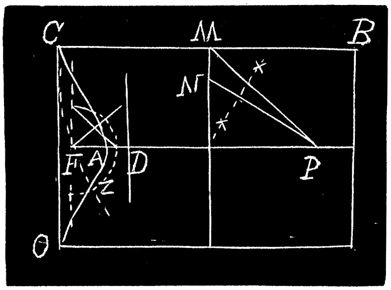

Fig. 2.

Lay off from M the distance $MP=a$. Then $O'P=\sqrt{(a^2-b^2)}$ is the distance from O' to the vertex. Lay off $O'N=O'P$. Then $NP=\sqrt{[2(a^2-b^2)]}$ is the distance from O' to the focus F, and $NQ=\frac{1}{2}NP$ is the distance from O' to the directrix. Draw a circle of any radius about F. Draw the chord of a quadrant and bisect it. The points of intersection of the circle and a line parallel to the directrix at a distance from it equal to the half chord lie on the hyperbola.

The applications of this problem are, of course, well known. For the sake of definiteness let us take the problem of finding the longest rectangle of a given width which can be laid on the floor of a rectangular room. If in Fig. 2 we draw a circle of radius $2a$ about B, it will cut the hyperbola at C and again at a point Z_1. If h is equal to or greater than OZ_1, BZ will obviously be equal to or less than $2a$. Hence if the rectangle has a width between OZ_1 and $2a$, it should be laid lengthwise of the room and can have a length $2a$. If its width is less than OZ_1, it should be "inscribed." Its length, BZ, will be greater than $2a$.

Fig. 2 is constructed for a rectangle 12×16 cm. It will obviously serve for any similar rectangle by proper choice of scale. OZ_1 measures 4.4+ cm.* Hence on the floor of a room 12′×16′ rectangles of width approximately 4′.4 or wider can be but 16′ long. Narrower rectangles can be longer. Measurement gives the length of one 3′ wide to be 17.2′+.*

NOTE ON LAMBERT'S METHOD OF SOLVING LINEAR DIFFERENTIAL EQUATIONS.

By **H. L. SLOBIN**, University of Minnesota.

In the July, 1910, *Annals*, Professor Lambert gives Cauchy's Method of Solving Linear Differential Equations, by breaking up the function

$$f\left(x,\ y,\ \frac{dy}{dx},\ \frac{d^2y}{dx^2},\ \dots,\ \frac{d^ny}{dx^n}\right)=0\dots(1),$$

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* Actual computation gives these values to four places of decimals to be: $OZ_1=4.4165$ and $BZ=17.2037$.

into *two* parts, f_1 and f_2, and introducing a parameter t so that $f=0$ is replaced by $f_1+tf_2=0\ldots(2)$.

He then assumes as a solution of (2),

$$y=A+Bt+Ct^2+\ldots,\ \ldots(3),$$

where A, B, C, ... are undeterminate functions of x, and substituting the expression (3) for y in (2) he equates to zero the coefficients of the successive powers of t, and solves the *differential equations* thus obtained for A, B, C, ...

Substituting these values obtained for A, B, C, ... in (3), and replacing it by unity, a series for y results, which, if convergent, is a solution of the differential equation.

It might be worthy of note that since the process is purely formal, and since t is arbitrarily introduced, in the end to be replaced by unity, we may break up the function into *several parts,* and introduce *various powers of* t, and then proceeding exactly as stated in Lambert's paper, the work for determining A, B, ... may frequently be much simplified; and where only *particular* solutions are desired, *the differential equations that must be solved to determine* A, B, ... *may be of lower degree* than the original equation. The aim of course is to obtain A, B, C, ... in the simplest form and with least difficulty.

Each particular example will readily suggest what powers of t should be introduced with the various parts into which f was broken up.

I illustrate my suggestion by applying it to the two examples used by Professor Lambert.

I. *For general solution:*

$$\frac{d^2y}{dx^2}+ax^2y=1+x\ldots(1).$$

$$\frac{d^2y}{dx^2}-1.t^2-xt^3+ax^2yt^4=0\ldots(2).$$

$$y=A+Bt+Ct^2+\ldots\quad\ldots(3).$$

$$\frac{d^2A}{dx^2}\Bigg| +\frac{d^2B}{dx^2}\Bigg| t+\begin{array}{r}\dfrac{d^2C}{dx^2}\\ -1\end{array}\Bigg| t^2+\begin{array}{r}\dfrac{d^2D}{dx^2}\\ -x\end{array}\Bigg| t^3+\begin{array}{r}\dfrac{d^2E}{dx^2}\\ +ax^2A\end{array}\Bigg| \begin{array}{l}t^4+\ldots\\ +\ldots\end{array}\equiv 0.$$

Equating to zero the coefficients of the successive powers of t, using only particular solutions in every case, since we only need two independent constants in the general solution, we have immediately, by inspection,

$$A=c_1,\ B=c_2x,\ C=\frac{x^2}{2\,!},\ D=\frac{x^3}{2.3},\ E=-aC_1\frac{x^4}{3.4},\ \text{etc.}$$

where the individual terms are exactly the same as in Lambert's solution, but where A, B, ... are monomials. Lambert shows that the series for y thus obtained is the sum of four convergent series, and that $y=A+B+C+$ ... is a solution.

II. *For a particular solution:*

$$x^2\frac{d^2y}{dx^2}+(x+2x^2)\frac{dy}{dx}-4y=0\ldots(1),$$

$$2x^2\frac{dy}{dx}+\left(x\frac{dy}{dx}-4y\right)t+x^2\frac{d^2y}{dx^2}t^3=0\ldots(2),$$

$$y=A+Bt+Ct^2+\ldots\ \ldots(3).$$

$$\begin{array}{l|l|l|l|l|l}
2x^2\frac{dA}{dx} & +2x^2\frac{dB}{dx} & t+2x^2\frac{dC}{dx} & t^2+2x^2\frac{dD}{dx} & t^3+2x^2\frac{dE}{dx} & t^4+\ldots \\
 & +x\frac{dA}{dx} & +x\frac{dB}{dx} & +x\frac{dC}{dx} & +x\frac{dD}{dx} & \equiv 0. \\
 & -4A & -4B & -4C & -4D & \\
 & & & +x^2\frac{d^2A}{dx^2} & +x^2\frac{d^2B}{dx^2} & +\ldots
\end{array}$$

Hence, $A=c_1$, $B=-2c_1x^{-1}$, $C=\frac{5}{2}c_1x^{-2}$, $D=-\frac{5}{2}c_1x^{-3}$, ..., etc., and series for y is obtained, which, if convergent, is a particular solution.

When by a different grouping of terms another independent particular solution can be obtained, the sum of the two will be the general solution. Of course, if the *general* solution is to be determined *directly*, the differential equations determining A, B, ... can not be of lower degree than the original differential equation, but as Lambert points out they are very much simpler than the given differential equation.